MATH-325 Group Theory-I

Credit hours: 3-0

Prerequisite: None

Course Objectives: This course aims to introduce students to the basic concepts of group theory. Algebra, developed by the Muslims, started as generalized arithmetic but went on to deal with linear, quadratic, cubic and quartic equations and systems of such equations. Later, Galois and Abel developed group theory to prove that there is no canonical solution of higher order polynomial equations by means of radicals. Two further branches of development followed the solution of simultaneous linear equations (leading to linear algebra) and the more formal structure of groups and their extensions (leading to rings and fields). This course presents the basic concepts of group theory.

Detailed Course Contents: Sets and relations, partitions and equivalence relations, binaryoperations, group, dihedral group, quaternion group, group of n^{th} roots of unity, group of residues, general linear group, subgroups, order of a group, order of an element of a group, cyclic groups, cyclic subgroups, generating sets, permutation, group of permutations, cycles,transpositions, even and odd permutations, alternating groups, decomposition of a permutation in to disjoint cycles, cosets, index of a subgroup in a group, Lagrange's theorem, consequences of Lagrange's theorem, normal subgroups, product of subgroups, group homomorphism, properties of a homomorphism, kernel of a homomorphism, isomorphism, factor groups, the first isomorphism theorem, center subgroup.

Learning Outcomes: On successful completion of this course, students will know

- Equivalence relations on sets, equivalence classes.
- Binary operation, group, subgroup, order of a group
- Dihedral group, quaternion group, group of n^{th} roots of unity, group of residues, generallinear group.
- Permutation, group of permutations, cycles, transpositions, even and odd permutations.
- Alternating group, decomposition of a permutation into disjoint cycles.
- Coset, index of a subgroup in a group, Lagrange's theorem.
- Normal subgroups, centre subgroup, commutator subgroup.
- Group homomorphism, kernel of a homomorphism, isomorphism.
- Factor group, the first isomorphism theorem.
- Product of subgroups.

Text book: J. A. Gallian, Contemporary Abstract Algebra, 8th ed. Brooks/Cole, CA, 2013.

Reference book:

1. W. Keith Nicholson, Introduction to Abstract Algebra, (3^{rd} edition), 2007, John Wiley &sons.
2. J. H. Fraleigh: A first course in abstract algebra (7^{th} edition), 1998, Addison-Wesleypublishing.
3. N. Herstein, Abstract Algebra, third edition,1995, Prentice Hall.

Weekly Breakdown		
Week	***Section***	***Topics***
1	Ch. 0	Equivalence relations, partition, equivalence classes partition, examples and related results. Functions, types of functions and composition of functions.
2	Ch. 1,2	Binary operations, groups, definitions and examples. Group of integers modulo n, dihedral group.
3	Ch. 2	Group of quaternion, group of n^{th} roots of unity, general linear group. Elementary properties of groups.
4	Ch. 3	Subgroups, order of a group, order of an element of a group, subgroup tests,cyclic subgroups, subgroup generated by a subset of a group. The center subgroup, entralizer and normalizer subgroups.
5	Ch. 4	Cyclic groups, Properties of the cyclic groups, related theorems, examples,generators of finite cyclic groups, generators of Z_n.
6	Ch. 5	Permutation of a set, permutation group of a set, symmetric group S_n, cycle notation, product of disjoint cycles, product of disjoint cycles commute.
7	Ch. 5	Order of a permutation, transpositons (2-cycles) and related theorems, even and odd permutations, the alternating group of degree n.
8	Ch. 7	Cosets, properties of cosets, index of a subgroup in a group, definition examples related results, and the theorem of Lagrange.
9	**Mid Semester Exam**	
10	Ch. 7	Consequences of Lagrange's theorem, converse of the Lagrange theorem, Product of subgroups.
11	Ch. 9	Definition examples of direct product of groups. Normal subgroups, definitions, examples and related results.
12	Ch. 9	Factor groups, examples and related theorems.
13	Ch. 10	Group homomorphism, Properties of a homomorphism, kernel of a homomorphism, definitions, examples and related results.
14	Ch. 10	Properties of elements under homomorphism, properties of subgroups under homomorphism, kernels are normal.
15	Ch. 6,10	Normal subgroups are kernels. Isomorphism of groups. Cayley's theorem.
16	Ch. 10	The isomorphism theorems for groups, applications.
17		Review
18	**End Semester Exam**	